Digital Adaptive Flight Control System for Aerospace Vehicles

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The application of digital adaptive control techniques to flight control of an aerospace vehicle is presented. A digital adaptive flight control system is described which uses identification and synthesis processes to determine what changes in control parameters are required to adapt to changes in the dynamics of an aerospace vehicle as it experiences a wide range of flight conditions. Identification of vehicle dynamics is accomplished by an improved identification technique that uses correlation of control signals caused by input fluctuations and does not require special test inputs that may disturb normal system operation. Synthesis of digital compensation to provide stabilization and desired response characteristics is accomplished by means of an analytical synthesis technique that determines the parameters of a digital compensator from information about the vehicle dynamics obtained by the identification technique and from performance criteria established by the system designer. A digital computer simulation of a digital adaptive pitch rate flight control system was performed which included identification of a fourth-order aerospace vehicle and actuation system and synthesis of a fourth-order digital compensator.

Nomenclature

I = pilot input
R = output of input prefilter
C = system output, including measurement noise
C' = system output
X = output of data prefilter
M = compensator output
N_1 = turbulence input
N_2 = measurement and quantization noise
$A(s)$ = response model transfer function
$B(z)$ = input prefilter transfer function
$F(z)$ = data prefilter transfer function
$D(z)$ = digital compensator transfer function
$G_A(s)$ = actuator transfer function
$G_V(s)$ = vehicle transfer function
$G_I(s)$ = pilot input filter transfer function
$G_T(s)$ = turbulence filter transfer function
$\phi_{xc}{}^k(i,j)$ = crosscorrelation of x_{k-i} and c_{k-j} at the kth sampling instant
ρ = correlation factor
a_i,b_i,c_i,d_i = coefficients of open-loop transfer functions
e_i = denominator coefficients of closed-loop transfer function

Introduction

In recent years a considerable amount of research has been conducted on adaptive control systems that use identification and synthesis techniques to determine what changes in control parameters are needed to compensate for changing dynamics of a controlled process. Some investigators who have accomplished important work in this field are listed in Refs. 1–5. This paper presents the results of a study to determine the feasibility of applying this type of adaptive control system to flight control of an aerospace vehicle. The results reported include improvements of the techniques used by previous investigators and the outcome of a digital computer simulation. Although the emphasis in this study was on adaptive flight control of an aerospace vehicle, the principles developed are general and may be applied to a variety of control situations.

The adaptive flight control system studied is shown in Fig. 1. The essential elements of the system include the controlled vehicle and actuation system, a digital compensator that accomplishes the direct control function, a response model that establishes the desired response of the vehicle to command inputs, an identification process that determines the pulse transfer function of the vehicle, and a synthesis process that determines the parameters of the digital compensator and input prefilter. The input prefilter modifies the inputs to the control system to cancel out the effects of inner-loop dynamics insofar as possible. The data prefilter modifies the input data used in the identification calculations to make possible a better identification in the presence of noise. The compensation, input shaping, data filtering, identification, and synthesis are done by digital computation. The analog portions of the system are the vehicle with its associated actuation system, the motion sensors, and the guidance and/or pilot command input transducers.

The identification technique uses correlation calculations to determine the pulse transfer function of the vehicle. This technique operates on the control signals resulting from input fluctuations and does not require test inputs. Except for the use of the data prefilter, which improves accuracy of identification in the presence of noise, this identification technique is essentially similar to that described in Ref. 3.

The synthesis technique used in the adaptive system provides a systematic analytical procedure for determining the parameters of the digital compensator from the pulse transfer function of the vehicle determined by the identification technique and from performance constraints established by the system designer. The types of performance criteria to which the synthesis technique is especially suited include constraints upon the locations of the closed-loop poles of the system and specification of position, velocity, and acceleration error constants. The synthesis technique was developed in this study and appears to be a more powerful technique than those proposed by previous investigators.[6,7]

From a mathematical point of view, the adaptive system is rather complicated. It is time varying and includes two interacting loops for direct control of vehicle and for adjusting the control parameters. The analysis in this paper assumes that the interaction between the two loops and the changes in the dynamics of the vehicle take place slowly enough that a linear time-invariant approach is valid. To confirm this, the digital computer simulation was carried out. The simulated system included identification of a fourth-order vehicle and actuation system and synthesis of a fourth-order digital compensator. The results of the simulation study indicate the feasibility of this type of adaptive system for flight control within limits of the speed with which vehicle dynamics may

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change and the relative level of the noise compared with the level of input fluctuations.

Identification

The basic problem of identification is to obtain a useful measure of the dynamics of the controlled vehicle and actuation system in the presence of noise. To accomplish this without using test signals that may interfere with normal system operation, correlation techniques are used.

Consider that the controlled vehicle and actuation system shown in Fig. 1 is a linear process of order N. It then may be described at sampling instants by a pulse transfer function (z transform) as given in Eq. (1):

$$G(z) = \frac{C(z)}{M(z)} = \frac{a_0 z^0 + \ldots + a_M z^{-M}}{b_0 z^0 + \ldots + b_N z^{-N}} \tag{1}$$

In Eq. (1) the coefficients a_i and b_i completely define the dynamics at sampling instants. The coefficient a_0 may be set equal to zero because physical systems do not respond in zero time, and the coefficient b_0 may be set equal to 1 without loss of generality. Making these substitutions, crossmultiplying, and regrouping terms,

$$C(z) = (a_1 z^{-1} + \ldots + a_M z^{-M})M(z) - (b_1 z^{-1} + \ldots + b_N z^{-N})C(z) \tag{2}$$

Using the fact that z^{-j} corresponds to a time delay of j samples, the inverse z transform of Eq. (2) is

$$c_j = a_1 m_{j-1} + \ldots + a_M m_{j-M} - b_1 c_{j-1} - \ldots - b_N c_{j-N} \tag{3}$$

where c_j is the value of c at the jth sampling instant. Multiplying by the input x_i and averaging yields

$$\overline{x_i c_j} = a_1 \overline{x_i m_{j-1}} + \ldots + a_M \overline{x_i m_{j-M}} - b_1 \overline{x_i c_{j-1}} - \ldots - b_N \overline{x_i c_{j-N}} \tag{4}$$

or in terms of correlation functions

$$\phi_{xc}(i,j) = a_1 \phi_{xm}(i,j-1) + \ldots + a_M \phi_{xm}(i,j-M) - b_1(\phi_{xc}(i,j-1) - \ldots - b_N \phi_{xc}(i,j-N) \tag{5}$$

By selecting $M + N$ values of i, Eq. (5) may be made to yield $M + N$ simultaneous equations that may be solved for the $M + N$ unknown coefficients. These are presented in matrix form in Eq. (6), where i is allowed to take on the integral values $j - 1$ to $j - M - N$, inclusive:

Equation (7) may be used to calculate the correlation functions on a continuous basis that weights current data more heavily than past data. The expression $\phi_{xc}{}^k(i,j)$ is the value of $\phi_{xc}(i,j)$ at the kth sampling instant,

$$\phi_{xc}{}^k(i,j) = \rho\phi_{xc}{}^{k-1}(i,j) + (1 - \rho)x_{k-i}c_{k-j} \qquad 0 < \rho < 1 \tag{7}$$

The constant ρ in Eq. (7) determines, essentially, the length of the memory with respect to past input data.

It has been shown[8] that the mean-squared-error in correlation functions calculated from noisy data is approximately proportional to the mean-squared-noise present and inversely proportional to correlation time. Important sources of noise to be considered are measurement noise, quantization noise, and turbulence.

Measurement noise is introduced mainly by instrument inaccuracies. Inaccurate data caused by instrument dynamic effects can be eliminated by proper choice of instruments. Random measurement noise will be eliminated by the correlation process upon which the identification technique is based. A systematic error introduced by zero offset can be eliminated by passing data through identical high-pass filters.[1]

Quantization noise results from analog/digital conversion and from round-off error in the computer. If a sufficiently small quantization level is used, then quantization error also may be treated like random noise.[9]

Turbulence, in effect, acts as extraneous noise. A sizable amount of experimental work has been done to determine the statistical character of atmospheric turbulence. There is considerable agreement that it may be represented by white noise passed through a low-pass filter where the filter time constant is a function of the speed of the vehicle, and the amplitude has been determined by experimental observations.

Circulating noise presents a special problem. Any noise entering the system shown in Fig. 1 within the feedback loop circulates so that every quantity around the loop has some component of the noise. The correlation process used in identification only discriminates against noise if the noise components associated with the quantities being correlated are unrelated. This is why in the derivation of the identification technique the inner-loop m and c data were correlated with the outer-loop x data. Discrimination against noise can be improved by properly relating x to m and c. In the digital simulation carried out in this study, the data prefilter $F(z)$ was added as a refinement to the basic identification technique. Its characteristics were made approximately

$$\begin{bmatrix} a_1 \\ \cdot \\ \cdot \\ \cdot \\ a_M \\ -b_1 \\ \cdot \\ \cdot \\ -b_N \end{bmatrix} = \begin{bmatrix} \phi_{xm}(j-1,j-1) \ldots \phi_{xm} & (j = 1, j - M) & \phi_{xc}(j-1,j-1) \ldots \phi_{xc} & (j-1,j-N) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{xm}(j-M-N,j-1) \ldots & \phi_{xm}(j-M-N,j-M) & \phi_{xc}(j-M-N,j-1) \ldots & \phi_{xc}(j-M-N,j-N) \end{bmatrix}^{-1} \times \begin{bmatrix} \phi_{xc}(j-l,j) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi_{xc}(j-M-N,j) \end{bmatrix} \tag{6}$$

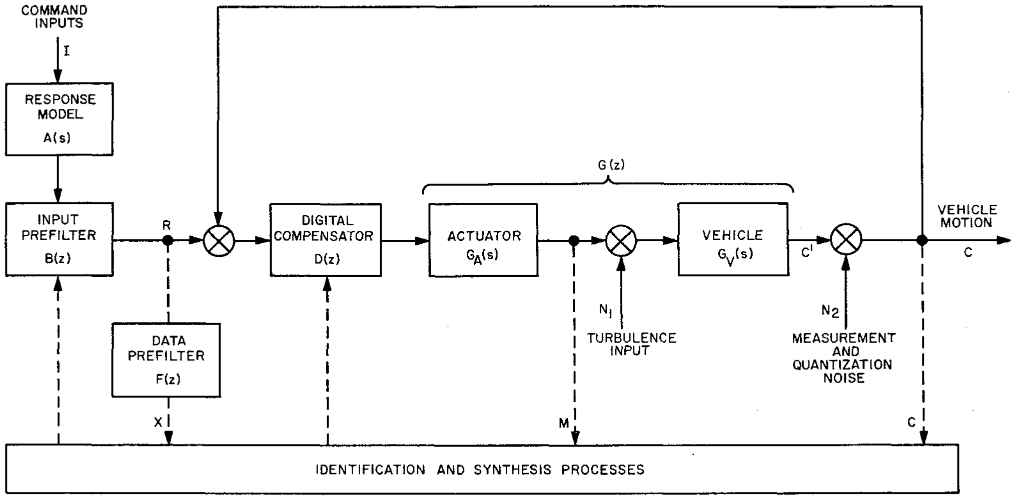

Fig. 1 Digital adaptive flight control system

equal to

$$F(z) = \frac{D(z)}{1 + D(z)G(z)} \tag{8}$$

which improves identification by essentially strengthening the correlation of x with m.

Synthesis

The synthesis problem has two aspects: 1) the derivation of the digital compensator parameters to provide desired inner-loop response to disturbance inputs; and 2) the derivation of the prefilter parameters to insure that command inputs, modified by the response model, will be followed as closely as possible and thus provide desired outer-loop command response characteristics.

An analytical synthesis procedure was developed in this study to handle the first aspect of the problem in a more satisfactory way than other techniques known to the writers. The general form of this procedure is presented in Ref. 10. A specialized form used in this study may be derived, as follows, from Fig. 1. Assume first that the digital compensator to be designed must have an integration to provide zero steady-state error, that is, the denominator of $D(z)$ must contain a $(1 - z^{-1})$ factor. For the synthesis procedure, it is convenient to lump this known factor with the known (from the identification procedure) transfer function $G(z)$:

$$G'(z) = \frac{1}{(1 - z^{-1})} G(z) = \frac{\sum_{i=0}^{M} a_i z^{-i}}{\sum_{i=0}^{N+1} b_i' z^{-i}} \tag{9}$$

$$b_i' = b_i - b_{i-1} \tag{10}$$

$$D(z) = \frac{1}{(1 - z^{-1})} D'(z) \tag{11}$$

and

$$D'(z) = \frac{\sum_{j=0}^{P} c_j z^{-j}}{\sum_{j=0}^{Q} d_j z^{-j}} \tag{12}$$

Suppose that the closed-loop system comprised of $D'(z)$ and $G'(z)$ has U closed-loop poles that are defined by the coefficients e_i, and the locations of these poles are specified by the system designer to determine the inner-loop dynamics. Because the closed loop contains the same zeros as the open loop, the closed-loop transfer function may be written in the following form:

$$K(z) = \frac{C(z)}{R(z)} = \frac{\sum_{i=0}^{M} a_i z^{-i} \sum_{j=0}^{P} c_j z^{-j}}{\sum_{j=0}^{U} e_i z^{-i}} \tag{13}$$

The transfer function from input to error contains as its zeros all poles of the open loop and as its poles all poles of the closed loop. This may be expressed as follows:

$$\frac{E(z)}{R(z)} = 1 - K(z) = \frac{\sum_{i=0}^{N+1} b_i' z^{-i} \sum_{i=0}^{Q} d_j z^{-j}}{\sum_{i=0}^{U} e_i z^{-i}} \tag{14}$$

Adding Eqs. (13) and (14) to eliminate $K(z)$ and rearranging yields Eq. (15):

$$\sum_{i=0}^{M} \sum_{j=0}^{P} a_i c_j z^{-i-j} + \sum_{i=0}^{N+1} \sum_{j=0}^{Q} b_i' d_j z^{-i-j} = \sum_{i=0}^{U} e_i z^{-i} \tag{15}$$

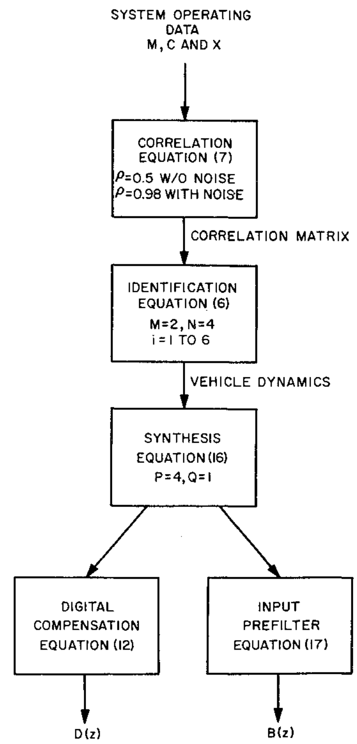

Fig. 2 Identification and synthesis processes

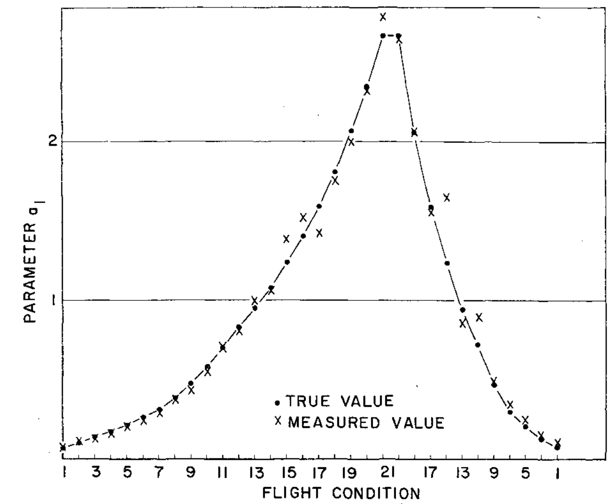

Fig. 3 Variation of a_1 during simulated mission while tracking six parameters simultaneously in the presence of noise

Expanding Eq. (15) and equating like powers of z yields a set of simultaneous linear algebraic equations which may be solved for the unknown compensator coefficients. The complete set of equations obtained in this manner forms the basic synthesis equation, which is expressed in matrix form in Eq. (16). To solve these equations, the values of P and Q must be chosen to obtain a square nonsingular matrix:

$$\begin{bmatrix} c_0 \\ \cdot \\ \cdot \\ \cdot \\ c_P \\ d_0 \\ \cdot \\ \cdot \\ \cdot \\ d_Q \end{bmatrix} = \begin{bmatrix} & & & & 1 & & & \\ a_1 & & & & b_1 & 1 & & \\ a_2 & a_1 & & & b_2 & b_1 & \cdot & \\ \cdot & a_2 & \cdot & \cdot & \cdot & b_2 & \cdot & 1 \\ \cdot & \cdot & \cdot & a_1 & \cdot & \cdot & \cdot & b_1 \\ \cdot & \cdot & \cdot & a_2 & \cdot & \cdot & \cdot & b_2 \\ a_M & \cdot & \cdot & \cdot & b_N & \cdot & \cdot & \cdot \\ & a_M & \cdot & \cdot & & b_N & \cdot & \cdot \\ & & \cdot & \cdot & & & \cdot & \cdot \\ & & & a_M & & & & b_N \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ e_1 \\ \cdot \\ \cdot \\ e_U \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \tag{16}$$

The second aspect of the problem, that of defining the parameters of the prefilter, is the simpler one. From the identification procedure $G(z)$ is known, and from the synthesis procedure $D(z)$ is known. The input prefilter is designed to cancel out insofar as possible the dynamics of the inner loop so that the system faithfully will follow the response model. This is accomplished by defining $B(z)$ as follows:

$$B(z) = z^{-1}\left[1 + \frac{1}{D(z)G(z)}\right] \tag{17}$$

where the z^{-1} term is required to insure realization.

Computer Simulation

The digital adaptive flight control system shown in Fig. 1 was simulated on a Univac Solid-State 90 computer. A pitch rate automatic pilot configuration was studied which provides stabilization of the vehicle and means for maneuvering in pitch through the introduction of pitch rate commands from the pilot. All calculations associated with optimum response shaping, adaptive control identification and synthesis, and digital compensation were carried out in the digital computer. For the study, both the actuation system and the vehicle also were simulated on the digital computer, and random input fluctuations representing the pilot command input, turbulence, and measurement and quantization noise were generated in the computer by a random number program.

The system configuration has the advantage that dynamic response to disturbance inputs and to pilot inputs may be adjusted separately. Inner-loop disturbance response is established by the characteristics of the digital compensator, which is adjusted adaptively to compensate for changes in vehicle dynamics. Outer-loop pitch rate maneuvering response is established by the response model, which defines the desired vehicle output, and the input prefilter, which shapes the input to the inner loop to compensate for varying inner-loop dynamics. The overall objectives to be met in the design of the digital adaptive flight control system were established as follows: 1) provide desired stabilization of vehicle and attenuation of pitch rate motion due to turbulence and other extraneous inputs; and 2) maintain desired pitch rate maneuvering characteristics throughout the entire range of flight conditions.

The transfer functions that represent the various portions of the system are given below. For the digital simulation, these were represented by equivalent difference equations. A sampling period of 0.1 sec was used. The vehicle transfer function was of the following form:

$$G_V(s) = \frac{K_V[(s/b) + 1]}{(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1} \tag{18}$$

In the simulation, a range of vehicle parameters was studied that included ω_n from 2 to 6 rad/sec, ζ from 0.3 to 0.7, b from 1.5 to 5, and K_V from 0.5 to 5. The response model and actuator were given by

$$A(s) = \frac{1}{(s^2/3^2) + (1.40/3)s + 1} \tag{19}$$

$$G_A(s) = \frac{1}{(s^2/21.2^2) + (1.41/21.2)s + 1} \tag{20}$$

The command input from the pilot was generated by filtering a random input with the filter $G_I(s)$, where

$$G_I(s) = \frac{K_I}{(s/0.3) + 1} \tag{21}$$

The turbulence input was generated by filtering a random input with the filter $G_T(s)$, where

$$G_T(s) = \frac{K_T}{(s/2.3) + 1} \tag{22}$$

The measurement and quantization noise inputs were generated by attenuating a random input by a gain that will be designated K_N.

The identification and synthesis computations associated with adaptive control are indicated in Fig. 2. Identification and synthesis was based on Eqs. (6, 7, 12, 16, and 17) presented previously. The desired system response to command inputs was established by the response model to be a natural frequency of 3 rad/sec and a damping ratio of 0.7. To help achieve this, the input prefilter characteristics were adjusted adaptively. The inner-loop design constraints are established by the coefficients e_i in the synthesis matrix. Two constraints were selected to provide a dominant inner-loop response mode having a natural frequency of 3.3 rad/sec and a damping ratio of 0.9. The remaining four were used to locate three inner-loop poles at 0.1 on the positive real axis in the z plane and to locate one pole at a_1/a_2 on the positive real axis in the z plane to cancel the vehicle zero and thus avoid a long tail in the disturbance response of the inner loop.

Two types of results were obtained: 1) operation of the digital adaptive system at a single flight condition with and without noise; and 2) operation with continuously varying flight conditions representative of a simulated boost-re-entry mission with and without noise (21 flight conditions were simulated). In runs without noise, an identification and

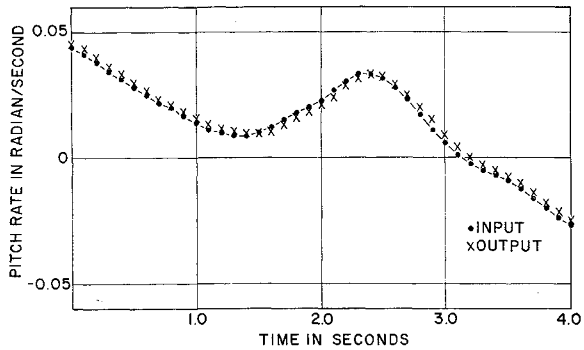

Fig. 4 Typical system response to random input

synthesis computation was performed every 2 sec to reoptimize the system. In runs with noise, a period of 10 sec was used so that noise would be correlated out more effectively. In runs with noise, the gains K_I, K_T, and K_N were adjusted so that the rms value of N_1 was 3% of the rms value of M, and the rms value of N_2 was 0.1% of the rms value of C'.

1. Operation at a Single Flight Condition

Simulation runs were performed to determine how fast the identification technique was capable of determining dynamics of the unknown process starting from a condition of complete ignorance. Without noise, this was accomplished consistently within 2 sec; with noise, this was accomplished consistently within 10 sec.

2. Operation with Continuously Changing Flight Conditions

The ability of the identification process to track changes in dynamics occurring in a simulated mission and of the synthesis process to produce suitable compensation is illustrated by Figs. 3–5. Figure 3 shows the tracking of one parameter out of six that were varied simultaneously during a simulated mission with noise. The other parameters tracked similarly. When noise was not present, the identified values and true values tracked more closely. Figure 4 shows the ability of the system to follow a random input. The results give evidence that the digital compensation designed by the adaptive system was capable of providing effective control. A final test performed on the digital adaptive system was to obtain step responses to command inputs with the compensator designs produced during the simulated mission. These are plotted in Fig. 5. The desired system step command response is given by the heavy line in this figure. The step responses obtained with the digital compensation designs produced during all runs without noise coincide with the desired response. The step responses obtained with the compensator designs produced during all runs with noise fall within the shaded area.

The digital computer requirements established by a system of the type described indicates the need for speeds of about 50,000 calculations/sec and about 4000 words of memory. This is within the capability of recently developed aerospace digital computers that are about $\frac{1}{4}$ to 1 ft^3 in physical size.

Conclusions

A digital adaptive flight control system has been described which uses identification and synthesis processes to determine what changes in control parameters are required to compensate for the varying dynamics of an aerospace vehicle in various regimes of flight. The performance of this system was studied in a digital simulation. The results of the simulation indicate that this type of adaptive control is feasible for flight control of an aerospace vehicle within limits on the

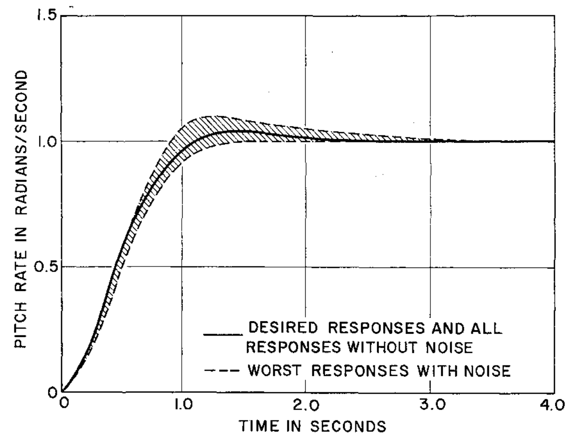

Fig. 5 System responses to step inputs

speed with which vehicle dynamics may change, and the relative level of the noise compared to the level of the input fluctuations also is indicated.

The digital simulation of the adaptive system indicated the severity of the noise problem in adaptive systems that use identification. In this regard, several improvements to increase the capability of the system described in this paper presently are being investigated. These improvements include the use of improved identification techniques as well as the use of a hybrid system that employs a simpler adaptive technique for making rapid rough parameter adjustments, which, in turn, are refined by the digital adaptive system at a slower rate. Evidence presently available indicates that a 100:1 improvement in the time required to discriminate against noise is feasible without difficulty.

It is becoming increasingly evident that recourse to digital computation will be a requirement as controls for advanced vehicles and other systems increase in complexity and refinement. The capabilities of aerospace digital computers have increased recently to the point where this can be considered seriously. The work reported in this paper represents a step toward the practical accomplishment of digital flight control and indicates possible directions of further research in this area.

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